

## **BUSINESS CYCLE AND SECTORAL FLUCTUATIONS: A NONLINEAR MODEL FOR CÔTE D'IVOIRE**

AKA<sup>1</sup>, Bédia F.

---

### **Abstract**

Although the share of service sector in Côte d'Ivoire's real GDP is higher than other sectors, it is widely recognized that the Ivorian economy is mainly based on agricultural sector and thus the fluctuations in this 'motor' sector could have a huge impact on the growth process of the country. We examine the issue of existence, identification and interaction of business cycle in Côte d'Ivoire's GDP and compare its fluctuations with the disaggregated main economic sectors (agriculture, industry and service), by using a univariate Markov regime switching model and its multivariate version over the period 1970-2001. We found similarities and simultaneity of business cycle between the sectors of the economy. While the real GDP's business cycle can be characterized, according to its mean duration (around 10 years) as a Juglar type cycle, the sectors' cycles mean duration is shorter from 4 to 5 years.

JEL Classification: C11, C22, E32

Keywords: Business cycles, Economic Growth, Markov switching, Structural breaks, Time series analysis.

---

### **1 Introduction**

The economic growth of Côte d'Ivoire is mainly based on exports of cocoa and coffee, and has been relatively high from 1960 to 1979 with the GDP per capita mean growth rate being around 5.7%. Noting that GDP could be approximately decomposed in sectoral value added, during this period qualified by observers as 'Miracle économique ivoirien', the shares of value added in GDP were 34% for agriculture, 15% for industry and 51% for services showing the importance of this last sector. This structure leads to growth in exports and increase in agricultural income and surplus managed by

---

<sup>1</sup> Department of Economics, University of Bouaké, Côte d'Ivoire. Email: [akbdia@yahoo.fr](mailto:akbdia@yahoo.fr)

CAISTAB<sup>2</sup> (a public marketing boards), and in investment in all economic sectors.

By the end of 1979, there was a slowdown in the growth process due to international shocks as the drop in international price of agricultural products and oil crises. During the 1980's the macroeconomic situation in Côte d'Ivoire was worsened and the government adopted Structural Adjustment Programs (SAP) financed by international institutional partners (mainly International Monetary Fund and World Bank) in order to improve the efficiency of the economy and enhance growth. The structural adjustment programs were combined with privatizations of public enterprises and liberalization policies at the beginning of 1990's. As if these macroeconomic policies results were not sufficiently growth improving, Côte-d'Ivoire's economy (including other Franc zone<sup>3</sup> countries) experienced the devaluation of the local currency 'CFA franc' in 1994. The combination of these various macroeconomic policies results unfortunately into negative impacts as growing unemployment and increase of poverty.

The current situation in Côte d'Ivoire is characterized by a social and political crisis since 1999 and mainly 2002, and no solution has yet been found to definitely overcome this crisis since the political 'Marcoussis agreements' in France analyzed by De Gaudusson (2003). Various other analyses have tried to understand and explain the origin of the crisis including Cogneau and Mesple-Somps (2003), Lepape (2003), Roubaud (2003), Hugon (2003) have shown that the crisis in Côte d'Ivoire has important economic and social impacts. The combination of these various policies could have generated different, similar or common cycle in the economic sectors' fluctuations and growth rate, which could have impacted the GDP growth rate diversely. It is therefore of interest to study the business cycle in the economic sectors of this economy and investigate new paths for growth.

---

<sup>2</sup> Caisse de Stabilisation et de Soutien du prix des produits agricoles.

<sup>3</sup> Including: Benin, Burkina Faso, Cameroon, Chad, Central African Republic, Côte-d'Ivoire, Gabon, Mali, Niger, Senegal and Togo

## **2. Ivorian business cycle and Economic Interactions**

The relative shares of sectors in Côte d'Ivoire's economy have evolved since 1960. Services, which represented 51% of the GDP, still remain predominant around 44% of GDP in 1998 while agriculture and industry which only represented 34% and 15% respectively in 1969 reach 28% for industry exceeding the share of agricultural value added in GDP in 1998.

The agricultural value added fell to 21.99% of GDP in 1999 but rose thereafter to 24.22% in 2000, 24.75% in 2001, 26.19% in 2002 and 27.65% in 2003. During the same period, industrial and services value added fell from 24.18% in 1999 to 20.77% in 2003 and from 53.83% in 1999 to 51.58% in 2003 respectively (*WDI database 2003*).

The economic structure describe above led to a growth in agricultural exports (total exports remained nearly constant around 43% of GDP on average from 1999 to 2003, and total imports represent 31% of GDP during the same period (*WDI database 2003*)) and revenues then managed by the CAISTAB (see Awudu and Philippe 2002, Fosu 1990a, Fosu 1990b). These revenues and the huge debt accumulated enabled the government to undertake various investment programmes in all sectors of the economy generating diverse growth rate across sectors. Total investments, which represented more than 15% of GDP and grew at a rate of 20% on average over the period 1960-1979 was 13.12% of GDP in 1999 and decreased steadily to 10.12% of GDP in 2003.

The change in the structure of Côte d'Ivoire economy induces thus an interest in investigating the co-movement and sources of fluctuations across Côte d'Ivoire economic sectors and in real GDP in line of recent theoretical and empirical business cycle research.

Business cycles analysis has used various methods since the pioneering work of Burns and Mitchell (1946). However, most of the contributions are concerned with linear<sup>4</sup> processes of time series and

---

<sup>4</sup> Among them the ARMA and ARIMA models of Beveridge and Nelson (1981), the unobservable components models of Harvey (1985), Watson

failed to explain business cycles features about duration of recessions and expansions.

In 1989 Hamilton proposed an innovative more general non-linear model called Markov regime-switching model (MRSRM) to analyse economic fluctuations, which has been widely used in recent years on quarterly or monthly data in developed economies by several authors (see Clements and Krolzig (2003), Krolzig and Toro (1999 and 2001), Krolzig (2001)) and thus as point out by Holmes (2003) few application exists for African countries where only short-range yearly data are available.

This paper uses the modern version of business cycles analysis, the Markov regime-switching model (MRSRM) to investigate the issues of existence and identification of business cycle in Côte d'Ivoire economic sectors. The univariate model of Hamilton (1989) was applied to quarterly USA GDP growth rate. This method has been recently generalized to multivariate MS-VAR model (see Krolzig 1997) allowing modeling several series subject to regime change. The MS-VAR model provides a useful methodological tool for extracting the common component of a group of economic time series and we use it here to address the issue of identifying the business cycles pattern in Côte d'Ivoire economic sectors. Following Krolzig (2000) we adopt a MS-VECM with two regime representing recession and expansion.

The remainder of the paper is organized as follows. Section 3 introduces the econometric methodology including the MS model innovated by Hamilton (1989) and the generalized multivariate version introduced by Krolzig (1997). Section 4 presents the results of estimation using the univariate model of the aggregate GDP and Section 5 presents the disaggregated multivariate model of sectors. Section 6 presents the analysis of interactions across sectors and with GDP and the last section concludes.

---

(1986), Clark (1987), the cointegration approach of King, Plosser, Stock and Watson (1991),

### 3. Econometric Methodology

In Hamilton's model (1989) of the US business cycle a  $p^{\text{th}}$ -order autoregression is fitted to the quarterly percent change in US real GNP from 1953 to 1984,

$$\Delta y_t - \mu(s_t) = \alpha_1(\Delta y_{t-1} - \mu(s_{t-1})) + \alpha_2(\Delta y_{t-2} - \mu(s_{t-2})) + \dots + \alpha_p(\Delta y_{t-p} - \mu(s_{t-p})) + u_t \quad (1)$$

$$u_t \sim NID(0, \sigma^2)$$

where  $\Delta y_t$  is 100 times the first difference of the log of real GNP and the conditional mean  $\mu(s_t)$  switches between two states,

$$\mu(s_t) = \begin{cases} \mu_2 > 0 & \text{if } s_t = 2 \text{ ('expansion', 'boom')} \\ \mu_1 < 0 & \text{if } s_t = 1 \text{ ('contraction', 'recession')} \end{cases} \quad (2)$$

and the variance  $\sigma^2$  is constant. The regime generating process is assumed to be a two-state hidden Markov chain such that  $p_{21}$  gives the probability of a transition from an expansion into a contraction and  $p_{12}$  denotes the probability of leaving the contraction state.

The initial approach developed by Hamilton of the US business cycle can be generalized to a multi-country model, as seen in the Annex. In the following the Hamilton's (1989) univariate two-regime model is first applied to each economic sector and to GDP over the period 1971-2001, and next we apply the multivariate version to the 3 sectors. Finally we introduce the GDP in the multivariate model to analyse the effects of sectoral shocks on GDP and other sectors as well.

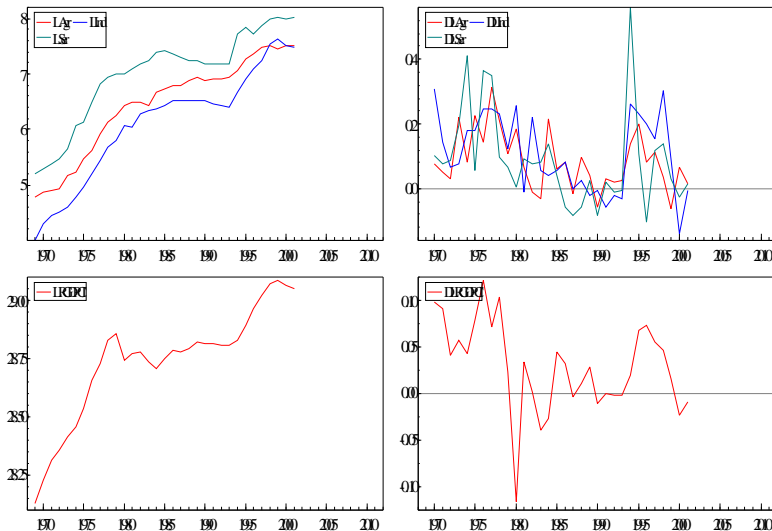
### 4. Aggregate GDP Results

The results are based on Côte-d'Ivoire's yearly real GDP and sectors' value added data (millions dollars at 1995 prices) from World Development Indicators database covering the period 1970-2001. We first proceed with the analysis of the stylised facts of business cycle in GDP and second we deal with each sector separately and compare the sector cycles to the GDP one. State 2 is related to expansion phase and state 1 to recession phase. In all cases

the variable  $y_t$  is computed as  $y_t = 100 * \ln(GDP_t / GDP_{t-1})$  for GDP, thus in the case of Markov switching autoregressive of order  $p$  models (MS-AR( $p$ )), the parameter  $\mu_0$  indicates the real GDP mean yearly growth rate in the contraction regime, and  $(\mu_0 + \mu_1)$  represents the mean yearly growth rate in the expansionary regime;  $p_{11}$  indicates the probability of staying in recession;  $p_{22}$  indicates the probability of staying in expansion;  $\rho = (1-q)/(2-q-p)$  indicates the probability to converge in long run toward a high growth rate;  $(1/(1-q))$  represents the expected duration of recession and  $(1/(1-p))$  the expected duration of expansion.

The yearly sector and real GDP growth rates are plotted in right panel of Figure 1 while left panel depicts the log level of the variables. From Figure 1 (left top panel) we can see that the level of service sector is higher compare to those of agricultural and industrial sectors, and there is more correlation between agricultural and industrial sectors. The results of the univariate model estimation are presented in Table 1.

**Figure 1: Log level and growth rate of variables**



**Table 1: EM Estimates (Univariate Model)**

	<b>Agriculture</b> MSM(2)- AR(2)	<b>Industry</b> MSM(2)- AR(2)	<b>Services</b> MSM(2)- AR(2)	<b>GDP</b> MSM(2)- AR(0)
$\mu_1$	2.71	1.15	3.56	0.0051
$\mu_2$	18.79	21.58	41.03	6.831
$\Phi_1$	-0.2626	0.0735	0.1658	
$\Phi_2$	-0.2371	-0.0457	-0.2841	
$\rho_{11}$	0.81	0.81	0.88	0.93
$\rho_{22}$	0.67	0.75	0.20	0.83
$\rho$	0.63	0.56	0.86	70.8
Observations	18.9	16.7	26	18.9
	11.1	13.3	4	12.1
Ergodic Prob.	0.63	0.56	0.86	0.71
	0.36	0.43	0.13	0.28
Duration	5.49	5.43	8.37	15.5
	3.11	4.13	1.26	6.21
ln-Likelihood	-105.79 (-107.0)	-107.81 (-112.5)	-113.30 (-123.1)	-87.54 (-91.8)
AIC	7.52	7.65	8.02	5.97
	(7.40)	(7.77)	(8.47)	(6.05)
HQ	7.62	7.75	8.12	6.05
	(7.46)	(7.83)	(8.53)	(6.08)

The computations were carried out using Markov switching and MS-VAR codes in Ox – PcGive (see Krolzig 1997).

Macroeconomic fluctuations in Côte d'Ivoire are marked by expansions dated by a MSM(2)-AR(0) model selected for the GDP as the periods from 1971 – 1978 and 1995 – 1998, with a mean growth rate of 6.84%. The economy is characterized by recession with mean growth rate of 0.005% from 1979 – 1994 and from 1999 to the end of the sample period 2001.

The mean duration of GDP' cycle is 10.8 years and the probability of the growth process to converge toward an expansion regime is 70.8% meaning that the country has a high potential of growth in future. Figure 2, in the Annex, depicts the probability of regimes in GDP. The first peak of GDP was in 1978 and the last peak in 1998.

## 5. Disaggregate GDP Results

In each sector case the log likelihood favours the non-linear MSM(2)-AR(2) model over the linear one. The services sector growth rate is higher than agriculture and industrial sector in expansion regime as well as in recession regime. The probability to converge in long run toward a high growth rate is 63% for agricultural sector, 56% for the industrial sector and 86% for the services sector, which is closer and even over the one of GDP 70.8%. Figure 2, in the Annex, depicts the probability of regimes in each sector.

Troughs of the sectors are coincident in 1993 and precede the GDP's trough one year in 1994 (see Table 2). The first peak of GDP was in 1978 while services peak in 1977 followed by the agricultural and industrial coincident peak in 1980. While the last peak of services and agriculture are in 1994 and 1995 respectively, the GDP last peak is in 1998 and coincident with industrial peak one year before the first coup. The mean duration of cycles computed from table 1 are 4.30 years for agriculture, 4.78 for industry and 5.31 years for services while the mean duration of GDP is 10.8 years.

The regime found for each sector suggests the use of the multivariate model described above to investigate a common cycle among sectors. The results of the multivariate model estimation are presented in Table 3. The log likelihood favours the non-linear MSIH(2)-VARX(2) model over the linear one.

**Table 2: Cycle Dating Comparisons**

	<b>Agriculture</b>	<b>Industry</b>	<b>Services</b>	<b>GDP</b>
Regime 1 (Recession)	1972 - 1972	1972 - 1973	1972 - 1973	1979 - 1994
	1981 - 1983	1981 - 1981	1975 - 1975	
	1985 - 1993	1983 - 1993	1978 - 1993	
	1996 - 2001	1999 - 2001	1995 - 2001	1999 - 2001
Regime 2 (Expansion)	1973 - <b>1980</b>	1974 - <b>1980</b>	1974 - 1974	1971 - <b>1978</b>
	1984 - 1984	1982 - 1982	1976 - <b>1977</b>	
	1994 - <b>1995</b>	1994 - <b>1998</b>	1994 - <b>1994</b>	1995 - <b>1998</b>

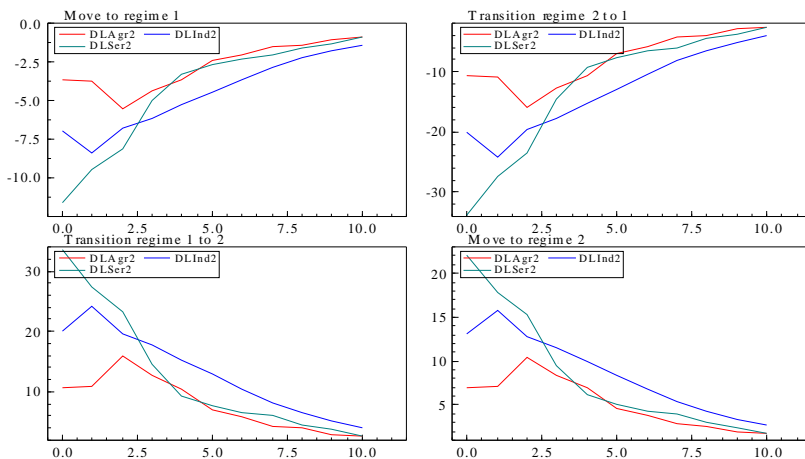
**Note:** Date of Peaks is in bold and date of Troughs is in *italic*.

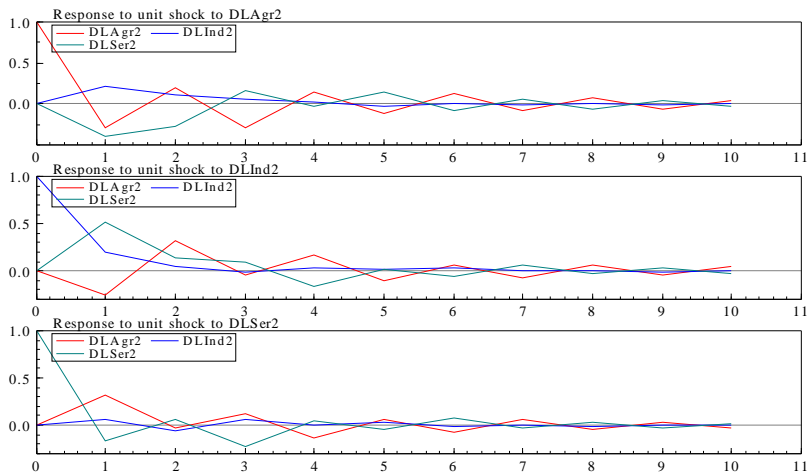


**Table 3: EM Estimates MSIH(2)-VARX(2)**

	Agriculture	Industry	Services
$\mu_1$	2.098	-0.168	3.805
$\mu_2$	12.733	19.961	37.466
$\Phi_1$	-0.2927	0.1914	-0.1732
$\Phi_2$	0.2898	0.0285	0.1185
$p_{11}$	0.9288		
$p_{22}$	0.8654		
Observations 16 ; 14; Ergodic Prob. 0.6541; 0.3459			
Duration 14.05; 7.43; ln-Likelihood -285.73; (-312.48)			
AIC 22.1822; (23.2322) HQ 22.8845; (23.7701)			

The services sector growth rates are higher than agriculture and industrial sector in expansion regime as well as in recession regime, confirming the results of the univariate model. The probabilities ( $p_{11}=0.92$ ) and ( $p_{22}=0.86$ ) indicate that the regimes are persistent meaning that the probability of remaining in a given regime is high. The sectoral path when there is a change in regime is traced in left panel of Figure 5, which shows that Côte d'Ivoire agricultural and service sectors move faster to recession (expansion) or from recession to expansion regime than industrial sector.

**Figure 5: Recovery of Sectors and Response to Sectoral Shock**




## 6. Interactions Analyses

The results of the multivariate model of the 3 sectors provide the following matrix  $P_1$  which shows the contemporaneous correlation between sectors. Bottom triangular panel presents the correlation in regime 1 while top triangular panel indicates correlation in regime 2. We can see that agricultural sector is negatively correlated with industrial and service sector in regime 1 while the industrial sector is positively correlated with service sector in that regime. On the other hand, there are positive correlations between all the 3 sectors of the economy in regime 2.

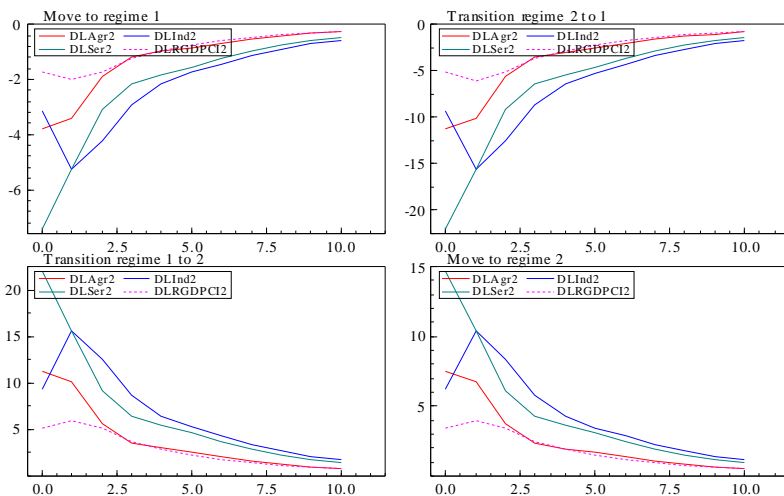
$$P_1 = \begin{pmatrix} AGR & 0.144 & 0.551 \\ -0.055 & IND & 0.277 \\ -0.294 & 0.706 & SER \end{pmatrix}$$

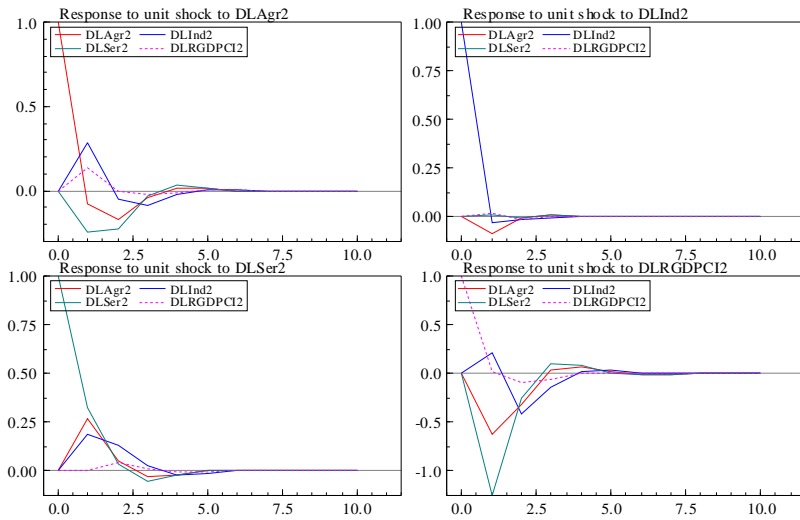
The response of the variables to a one-percentage innovation in each of the variables is in right panel of Figure 5 in the Annex. It could be seen that a one percent shock in agricultural sector leads to an increase in industrial sector but a decrease in service one period before growing after period two. A one percent shock in industrial sector leads to an increase in service one period before decreasing after period one but results in a decrease in industrial sector one

period before growing next periods. A one percent shock in service sector leads to an increase in agricultural and industrial sectors one period before decreasing after period one. While the short run effects are diversified the long run effects of a shock in one sector is positive on other sectors of the economy.

Including the real GDP in the estimation, the path when there is a change in regime is in left panel of Figure 6 while the response of the variables to a one-percentage innovation in each of the variables is in right panel of Figure 6.

**Figure 6: Recovery in Sectors and GDP and Response to Sectoral Shock**





It could be seen that agricultural sector move together with real GDP faster to recession (expansion) or from recession to expansion regime while industrial and services sectors move together more slowly to recession (expansion) or from recession to expansion regime. These facts confirm the preeminence of agricultural sector over industry and services sectors in the growth path of the economy.

The following matrix  $P_2$  shows the contemporaneous correlation between sectors and GDP. Lower triangular panel presents the correlation in regime 1 while upper triangular panel indicates correlation in regime 2. All sectors are negatively correlated with GDP in recession regime while the agricultural and industrial sectors are positively correlated with GDP in expansion regime but not service sector. Positive correlation with GDP in regime 2 is higher for agriculture than industry.

$$P_2 = \begin{pmatrix} AGR & -0.111 & 0.276 & 0.275 \\ 0.311 & IND & 0.418 & 0.049 \\ 0.002 & 0.546 & SER & -0.134 \\ -0.378 & -0.549 & -0.149 & GDP \end{pmatrix}$$

The responses of the variables to a one-percentage innovation in each of the variables depicted in right panel of Figure 6 show that a one percent chock in the agricultural sector leads to an increase in the industrial sector and in real GDP but a decrease in services one period before increasing. A one percent chock in the industrial sector leads to a slight increase in the service sector and real GDP one period before decreasing after period one but results in a decrease in the agricultural sector one period before increasing next periods. A one percent chock in the service sector leads to an increase in agricultural and industrial sectors one period before decreasing after period one and in a slight increase in real GDP. A shock in agricultural sector has a higher impact on GDP than other sectors in short run.

These results agree with the inter-sectoral studies by Guisan and Exposito(2001), (2005) and (2007), who emphasize the great positive importance of industry for economic development of services in Africa, and the positive effect of education and other factors related with human and social capital in order to increase industrial investment per capita.

## **7. Final Remarks**

This paper has investigated the issues of existence and identification of a business cycle in the 3 main sectors of Côte d'Ivoire economy using the Hamilton's (1989) univariate Markov regime-switching model, and the multivariate version of this model by Krolzig (1997, 2000). We found that the sectors' business cycle's mean duration, which is almost 4 to 5 years, suggests a Kitchin type cycle characterizing the sectors of the economy, while the real GDP of Côte d'Ivoire follows a Juglar type cycle (lasting 10 years) as found in Aka (2004). We found similarities and synchronization of business cycles pattern between sectors, and more importantly the path of agricultural sector business cycle appears to be closer to the real GDP's cycle. Although agricultural sector is the motor of economic growth in Côte d'Ivoire the services sector appears to have the highest probability of convergence (86%) toward an expansion regime followed by agriculture (63%) and industry (56%). The probabilities show that the different phases of sectors' cycle are persistent and thus any unexpected chock on a sector of the economy will permanently alter the future level of other sectors or real GDP.

The economic growth perspectives of the country are actually subject to a rapid resolution of the ongoing crisis as peace and safety environment are required conditions to attract foreign investment and international cooperation necessary to guarantee sectoral growth and development in Côte d'Ivoire.

## **References**

Aka BF, (2004) Do WAEMU Countries Exhibit a Business Cycle? A Simulated Markov Switching Model for a Western Africa Area. *Applied Econometrics and International Development*, vol. 4-4, pp. 43-58.

Awudu, A. and Philippe, J. (2002) Export and Economic Growth: Cointegration and Causality Evidence for Côte d'Ivoire. African Development Bank, pp. 1-17.

Beveridge S, Nelson C (1981) A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle. *Journal of Monetary Economics*, vol. 7, pp. 151-174.

Burns AF, Mitchell WC (1946) *Measuring Business Cycles*. New York: National Bureau of Economic Research.

Clark PK (1987) The cyclical component of U.S. economic activity. *Quarterly Journal of Economics*, vol. 102, pp. 797-814.

Clements MP, Krolzig H-M (2003) Business cycle asymmetries: characterization and testing based on Markov-switching autoregressions. *Journal of Business and Economic Statistics*, vol 21, pp. 196–211.

Cogneau, D. and Mesplé-Soms, S. (2003) Les illusions perdues de l'économie ivoirienne et la crise politique, *Afrique contemporaine*, vol. 206, pp. 87-103.

De Gaudusson, J.D., (2003) L'accord de Marcoussis, entre droit et politique, *Afrique contemporaine*, vol. 206, pp. 41-55.

Fosu, A.K. (1990a) Export Composition and the Impact of Exports on Economic Growth of Developing Economies. *Economic Letters*, vol. 34, pp. 67-71.

Fosu, A.K. (1990b) Export and Economic Growth: The African Case. *World Development*, vol. 18 (6), pp. 831-35.

- Guisan, M.C. and Exposito, P. (2001). Education, Industry, Trade and Development of African Countries in 1980-99. *Applied Econometrics and International Development* Vol. 2-2, pp.85-107.
- Guisan, M.C. and Exposito, P. (2005). Human Capital and Economic Development in Africa: An Econometric Analysis for 1950-2002. *Applied Econometrics and International Development*, Vol. 5-1, pp.129-142.
- Guisan, M.C. and Exposito, P.(2007). Education, Development and Health Expenditure in Africa: A cross-section model of 39 countries in 2000-2005. *Applied Econometrics and International Development*, Vol. 7-2, pp.135-142.
- Hamilton JD (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, vol. 57, pp. 357-384.
- Hamilton JD (1990) Analysis of time series subject to changes in regime. *Journal of Econometrics*, vol. 45, pp. 39-70.
- Hamilton JD (1991) A quasi-bayesian approach to estimating parameters for mixture of normal distributions. *Journal of Business and Economic Statistics*, vol. 9, pp. 27-39.
- Harvey AC (1985) Trend and cycles in macroeconomic time series. *Journal of Business and Economic Statistics*, vol. 3, pp. 216-227.
- Holmes, M.J. (2003) Are the Trade Deficits of Less Developed Countries Stationary? Evidence for African Countries. *Applied Econometrics and International Development*, vol. 3-3, pp. 7-24.
- Hugon, P. (2003) La Côte d'Ivoire, plusieurs lectures pour une crise annoncée. *Afrique contemporaine*, vol. 206, pp. 105-127.
- King, R.G., Plosser, C.I. Stock, J.H. and Watson, M.W. (1991) Stochastic Trend and Economic Fluctuations. *American Economic Review*, vol. 81, pp. 819-840.
- Kim C-J (1994) Dynamic linear models with Markov-switching. *Journal of Econometrics*, vol. 60, pp. 1-22.
- Krolzig H-M (1997a) Markov-switching models of the German business cycle. Mimeo, Institute of Economics and Statistics, University of Oxford.
- Krolzig H-M (1997b) Markov switching vector autoregressions. Modelling, statistical inference and application to business cycle analysis. Berlin: Springer.

Krolzig H-M (2001) Markov-switching procedure for dating Euro-zone business cycle. *Vierteljahreshefte zur Wirtschaftsforschung*, vol. 70, pp. 339-351.

Krolzig H.-M. (2001) Business cycle measurement in the presence of structural change: international evidence. *International Journal of Forecasting*, vol. 17, pp. 349-368.

Krolzig H-M, Toro J (1999) A new approach to the analysis of shocks and the cycle in a model of output and employment. European University Institute, EUI Working Paper ECO 99/30.

Krolzig H-M, Toro J (2001) Classical and modern business cycle measurement: The European case. University of Oxford, Discussion Paper in Economics 60.

Lepape, M. (2003) Les politiques d'affrontement en Côte d'Ivoire 1999-2003. *Afrique contemporaine*, vol. 206, pp. 29-39.

Roubaud, F. (2003) La crise vue d'en bas à Abidjan: ethnicité, gouvernance et démocratie. *Afrique contemporaine*, vol. 206, pp. 57-86.

Watson, M.W. (1986) Univariate Detrending Methods with Stochastic Trends. *Journal of Monetary Economics*, vol. 18, pp. 49-76.

Annex on line at the journal Website: <http://www.usc.es/economet/ijaeqs.htm>



## Annex

### A1. Econometric Methodology: Hamilton Model

In Hamilton's model (1989) of the US business cycle a  $p^{\text{th}}$ -order autoregression is fitted to the quarterly percent change in US real GNP from 1953 to 1984,

$$\Delta y_t - \mu(s_t) = \alpha_1(\Delta y_{t-1} - \mu(s_{t-1})) + \alpha_2(\Delta y_{t-2} - \mu(s_{t-2})) + \dots + \alpha_p(\Delta y_{t-p} - \mu(s_{t-p})) + u_t \quad (1)$$

$$u_t \sim NID(0, \sigma^2)$$

where  $\Delta y_t$  is 100 times the first difference of the log of real GNP and the conditional mean  $\mu(s_t)$  switches between two states,

$$\mu(s_t) = \begin{cases} \mu_2 > 0 & \text{if } s_t = 2 \text{ ('expansion', 'boom')} \\ \mu_1 < 0 & \text{if } s_t = 1 \text{ ('contraction', 'recession')} \end{cases} \quad (2)$$

and the variance  $\sigma^2$  is constant. The regime generating process is assumed to be a two-state hidden Markov chain such that  $p_{21}$  gives the probability of a transition from an expansion into a contraction and  $p_{12}$  denotes the probability of leaving the contraction state.

The initial approach developed by Hamilton of the US business cycle can be generalized to a multi-country model. For a given regime, the co-movements of growth rates are represented by a vector autoregressive model. Thus the international (from country  $m$  to country  $n$ ) and intertemporal (lag  $k$ ) transmission of country-specific shocks  $u_{mt}$  is determined by the autoregressive coefficients  $a_{nm,k}$  which are collected in matrices  $A_k = [a_{nm,k}]$ . Conditional on the state of the Franc zone economy  $s_t$ , the  $K$ -dimensional time series vector  $\Delta y_t = (\Delta y_{1t}, \dots, \Delta y_{Kt})'$ ,  $t = 1, \dots, T$ , is generated by a vector autoregression of order  $p$ ,

$$\Delta y_t - \mu(s_t) = A_1(\Delta y_{t-1} - \mu(s_{t-1})) + \dots + A_p(\Delta y_{t-p} - \mu(s_{t-p})) + u_t \quad (3)$$

where the presample values  $\Delta y_0, \dots, \Delta y_{1-p}$  are fixed. The innovation process  $u_t = \Delta y_t - E[\Delta y_t | Y_{t-1}, S_t]$  is a zero-mean white noise process with a variance-covariance matrix  $\Sigma$ , which is usually assumed to be Gaussian:  $u_t \sim \text{NID}(0, \Sigma)$ . Thus, the conditional expectation of  $\Delta y_t$  given its history  $Y_{t-1} = (\Delta y'_{t-1}, \dots, \Delta y'_{1-p})'$  and the regime vector  $S_t = (s_t, s'_{t-1}, \dots, s'_{t-p})'$  is determined by the system of  $K$  linear difference equations

$$E[\Delta y_t | Y_{t-1}] = \mu(s_t) + \sum_{k=1}^p A_k (\Delta y_{t-k} - \mu(s_{t-k})) \quad (4)$$

If the Franc zone economic growth process is subject to shifts in regime, the mean  $\mu$  (and possibly other parameters) of the linear time series model (3) vary with the state  $s_t$  of the business cycle. The general idea behind the class of regime-switching models is that the parameters of an autoregressive process are conditioned on an unobservable regime variable  $s_t \in \{1, \dots, M\}$ . The regime variable  $s_t$  denotes the unobserved state of the system (e.g. 'expansion' and 'contraction'). A Markov-switching vector autoregressive models of order  $p$  and  $M$  regimes is called the MS(M)-VAR(p) model.

In an MS-VAR model, the dynamic propagation mechanism of impulses to the system consists of (i) a linear autoregression representing the international transmission of national shocks and (ii) the regime shifts generating Markov process representing large – contemporaneously occurring – common shocks. These two sources of macro-economic fluctuations are not necessarily independent. If the variance is considered to be regime-dependent, the regime generating process alters the contemporaneous correlation of the innovations  $u_t$  and thus the orthogonalized impulse response function. In this sense, changes in regime can simultaneously affect

the state of the Franc zone business cycle and the international transmission of country-specific shocks.

In the following we will see that a major advantage of the MS-VAR model is its flexibility in modelling multiple time series subject to regime shifts (see Krolzing, 1997b). For example, in the most general specification of an MS-VAR model, all parameters of the autoregression could be conditioned on the state  $s_t$  of the Markov chain such that  $\mu(s_t), A_1(s_t), \dots, A_p(s_t), \Sigma(s_t)$  were parameter shift functions describing the dependence of the parameters  $\mu, A_1, \dots, A_p, \Sigma$  on the realized regime  $s_t$ . For purposes of business cycle analysis it is usually sufficient to consider regime shifts in the mean of the time series vector,

$$\mu(s_t) = \begin{cases} \mu_1 & \text{if } s_t = 1, \\ \vdots & \\ \mu_M & \text{if } s_t = M \end{cases} \quad (5)$$

The description of the data-generating process is not completed by the conditional process (3). Since the parameters depend on a regime, which is assumed to be stochastic and unobservable, a regime generating process for the states  $s_t$  has to be formulated. As in the Hamilton model, the regime generating process is an ergodic Markov chain with a finite number of states  $s_t = 1, \dots, M$  which is defined by the transition probabilities

$$p_{ij} = \Pr(s_{t+1} = j | s_t = i), \quad \sum_{j=1}^M p_{ij} = 1 \quad \forall i, j \in \{1, \dots, M\} \quad (6)$$

More precisely, it is assumed that  $s_t$  follows an ergodic  $M$  state Markov process with the irreducible transition matrix  $P$ ,

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{pmatrix} \quad (7)$$

where  $p_{iM} = 1 - p_{i1} - \cdots - p_{i,M-1}$  for  $i = 1, \dots, M$ . Using this law, the evolution of regimes can then be inferred from the data.

The maximization of the likelihood function of an MS-VAR model entails an iterative estimation technique to obtain estimates of the parameters of the autoregression and the transition probabilities governing the Markov chain of the unobserved states. Denote this parameter vector by  $\lambda$ , so that the Hamilton model  $\lambda = (\mu_s, \alpha_1, \dots, \alpha_4, \sigma^2, p_{11}, p_{22})$ .  $\lambda$  is chosen to maximize the likelihood for given observations  $Y_t$ .

Maximum likelihood (ML) estimation of the model is based on a version of the Expectation-Maximization (EM) algorithm discussed in Hamilton (1990) and Krolzig (1997b). The EM algorithm has been introduced by Dempster, Laird and Rubin (1977) for a general class of models where the observed time series depends on some unobservable stochastic variables – which are here the regime variables  $s_t$ . Each iteration of the EM algorithm consists of two steps: an expectation step and a maximization step. In the expectation step, the unobserved states  $s_t$  are estimated by their smoothed probabilities  $\Pr(s_t | Y_T)$ ; where all the conditional probabilities  $\Pr(s | Y, \lambda^{(j-1)})$  are calculated with the filtering and smoothing recursions using the estimated parameter vector  $\lambda$ . In the maximization step, an estimate of the parameter vector  $\lambda$  is derived as a solution  $\lambda^*$  of the first-order conditions associated with the likelihood function, where the unknown conditional regime probabilities  $\Pr(s | Y, \lambda)$  are replaced with the smoothed probabilities

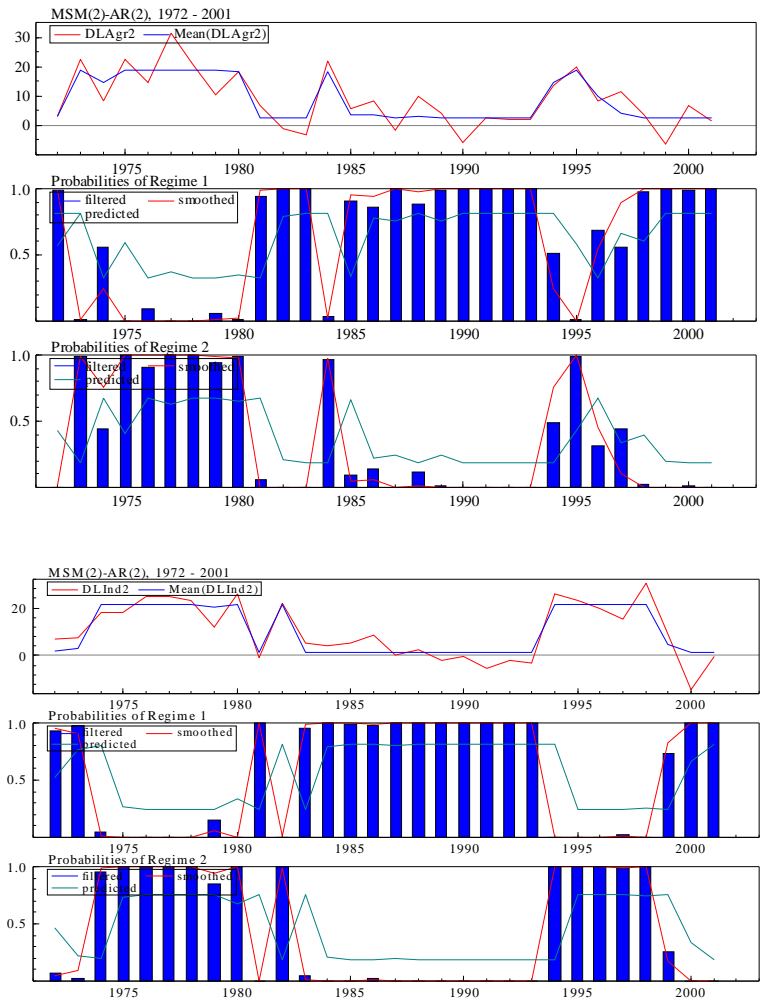
$\Pr(S|Y, \lambda^{(j-1)})$  derived in the last expectation step. Equipped with the new parameter vector  $\lambda$ , the filtered probabilities  $\Pr(s_t|Y_t, \lambda^{(j)})$  and smoothed probabilities  $\Pr(s_t|Y_t, \lambda^{(j)})$  are updated and so on. Thus, each EM iteration involves a pass through the filtering and smoothing algorithms, followed by an update of the first-order conditions and the parameter estimates guaranteeing an increase in the value of the likelihood function.

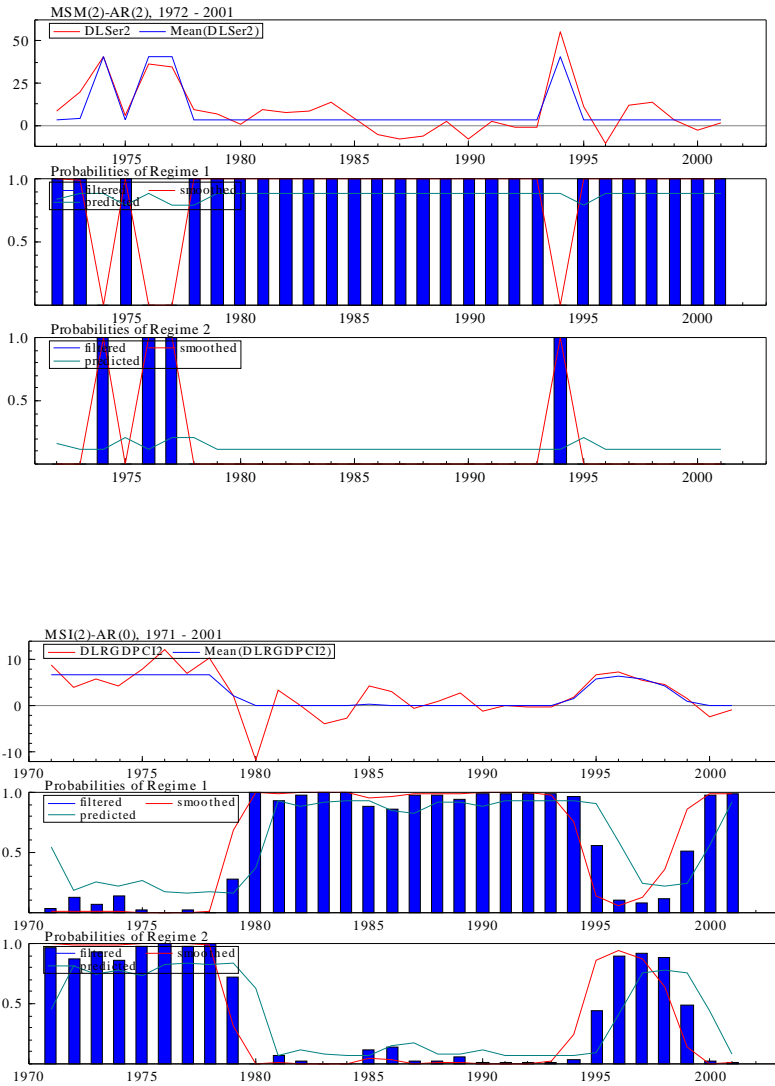
In conjunction with the filtering algorithm of Hamilton (1988, 1989) and the smoothing algorithms of Kim (1994), we can conduct optimal inference on the latent state of the economy by assigning probabilities to the unobserved regimes 'expansion' and 'contraction' conditional on the available information set. Regimes reconstructed in this way are an important instrument for the interpretation of MS-VAR models.

In this study the Hamilton's (1989) univariate two-regime model is first applied to each economic sector and to GDP over the period 1971-2001, and next we apply the multivariate version to the 3 sectors. Finally we introduce the GDP in the multivariate model to analyse the effects of sectoral shocks on GDP and other sectors as well.

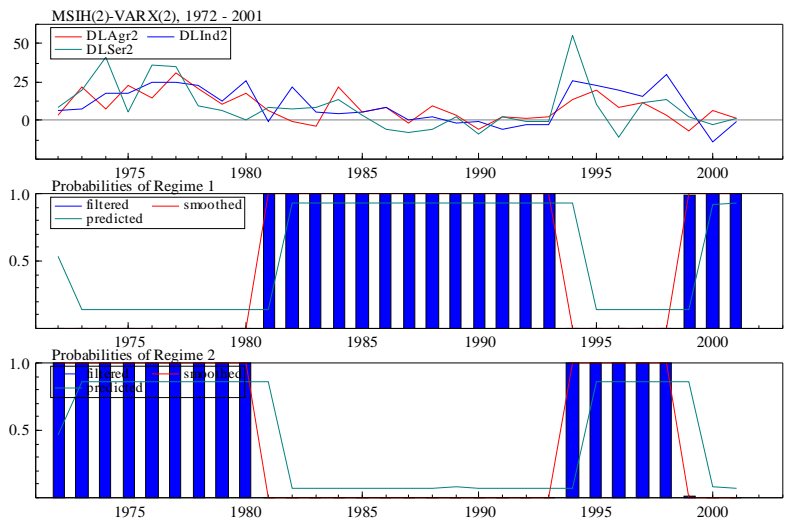
A2. Filter Probabilities

Figure 2: Filter Probabilities (MSM-AR(p))

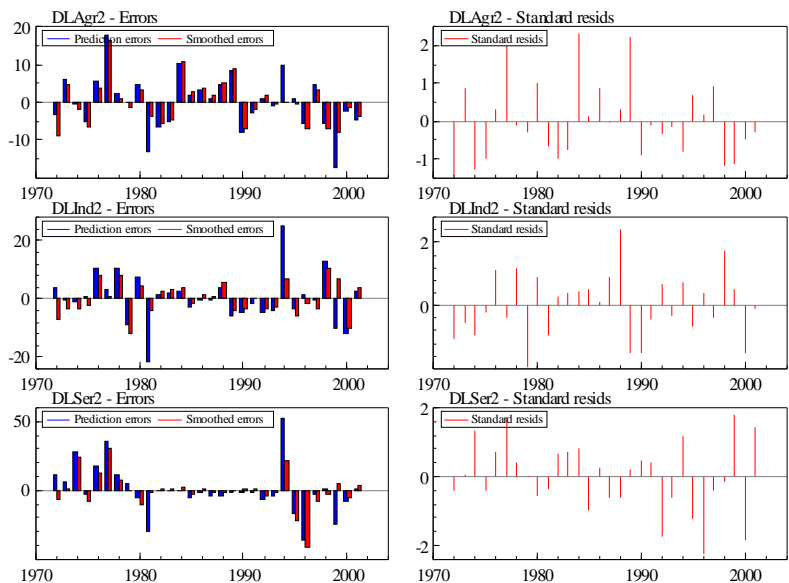




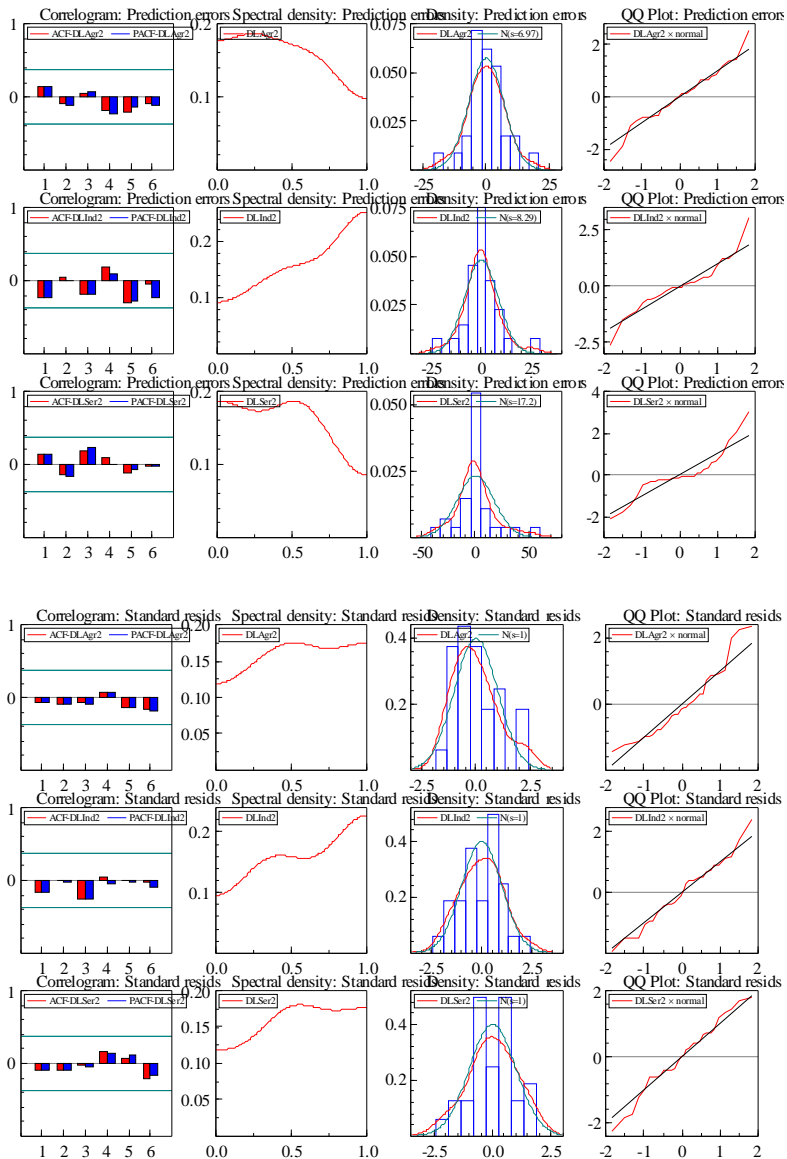
**Figure 3: Filter Probabilities and Smoothed and Predicted Errors in the MSIH(2)-VARX(2) Model**

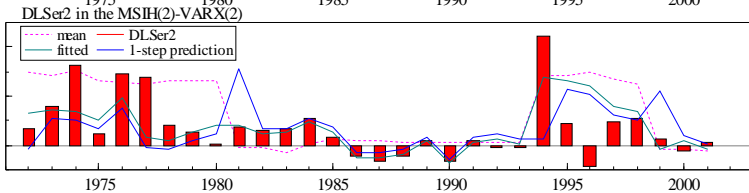
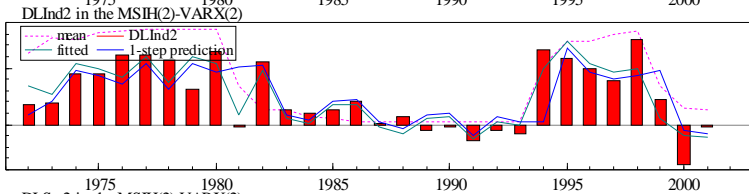
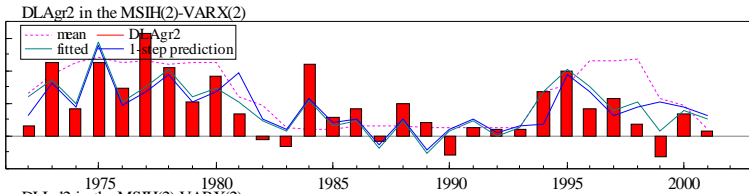
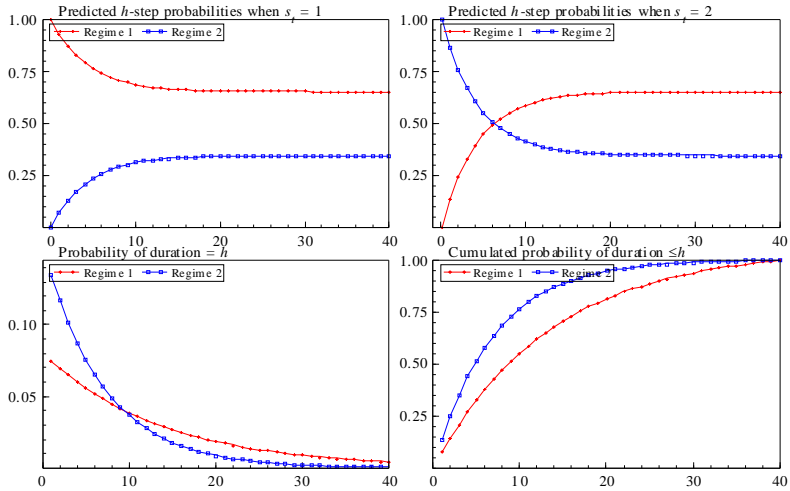






**Figure 4: Statistical Properties of the Smoothed and Predicted Errors**





## Annex 2. Response to Sectoral Shock

**Figure 6: Recovery in Sectors and GDP and Response to Sectoral Shock**

